# M. P. BIRLA FOUNDATION HIGHER SECONDARY SCHOOL
# SELECTION EXAMINATION 2020 - 21
# CLASS XII
# MATHEMATICS

***(Maximum Marks: 80)***
***(Time allowed: Three hours)***

(*Candidates are allowed additional 15 minutes for* ***only*** *reading the paper.)*

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*The Question Paper consists of three sections A, B and C.*
*Candidates are required to attempt all questions from* ***Section A*** *and all questions* ***EITHER*** *from* ***Section B OR Section C***
***Section A:*** *Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***
***Section B:*** *Internal choice has been provided in* ***one question of two marks and one question of four marks.***
***Section C:*** *Internal choice has been provided in* ***one question of two marks and one question of four marks.***
*All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.*
*The intended marks for questions or parts of questions are given in brackets [ ].*

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## SECTION-A (65 MARKS)

**Question 1.** **[15x1]**

In sub-parts (i) to (ix) choose the correct options and in sub-parts (x) to (xv), answer the questions as instructed.

(i). If the relation R on the set A = {1, 2, 3, 4, 5, 6} is defined as

R = { ( x, y) : x , y ∈ A , y = x + 1} , then

a.) (6 , 7) ∈ R

b.) (6 , 5) ∈ R

c.) (5 , 6) ∈ R

d.) (5 , 4) ∈ R

(ii). If A = { p , q } and B = { 1, 3 , 5 }, then the number of one - one functions from A to B is

a) 8

b) 0

c) 9

d) 6

(iii). Using principal values , the simplified value of $\tan^{-1} \tan \frac{7\pi}{6} + \sin^{-1} \sin \frac{\pi}{3}$ is

a) $\pi/2$

b) $\pi/6$

c) $3\pi/2$

d) $3\pi$

(iv). If $\begin{bmatrix} 6 & x \\ 2 & -1 \end{bmatrix}$ is a singular matrix, then the value of 'x' is

a) 3

b) -3

c) 0

d) none of the above

(v). If A is an invertible matrix of order 3 and $|A| = 5$ , then the value of $|\text{Adj } A|$ is

a) 20

b) 125

c) 5

d) 25

(vi). For what value of 'p' is $\begin{bmatrix} 7 & -2 \\ p & -4 \end{bmatrix}$ a symmetric matrix ?

a) - 2

b) - 4

c) 7

d) - 14

(vii). The point on the curve $y = x^2 - 2x + 7$ at which the tangent is parallel to the line $2x - y + 9 = 0$ is

a) ( - 2 , 15 )

b) ( 0 , 7)

c) ( 2 , 7 )

d) ( 2 , -7 )

(viii). The slope of the normal to the curve $y = x^3 - 5x^2 - x + 1$ at the point on it whose x - co- ordinate is 1 is

a) - 8

b) - 1/8

c) 1/8

d) 8

(ix). If A and B are two mutually exclusive events such that P(A) = ½ and $P(\overline{B}) = 2/3$ , then the value of $P(\overline{A} \cap \overline{B})$ is

a) 1/6

b) 5/6

c) 2/3

d) 1/3

(x). Find the absolute minimum value of the function $y = x^3$ in $[ - 2 , 2 ]$.

(xi). Let 'R' be the relation on Z (set of integers) defined by

$R = \{ ( a , b) : a , b \in Z , |a - b| \leq 5 \}$. Check whether R is reflexive.

(xii). If A is a square matrix of order 2 and $|A| = - 7$, then write the value of

$|3 A|$ .

(xiii). Find the degree of the differential equation

$$\left[ 1 + \left(\frac{dy}{dx}\right)^2 \right]^{4/3} = 5 \frac{d^2y}{dx^2}$$

(xiv). If A and B are events such that P (A) = 0.8 , P (B) = 0.5 and P( B/A ) = 0.4 ,

then find the value of $P ( A \cap B )$ .

(xv). Two balls are drawn from an urn containing 2 white, 3 red, and 4 black balls

one by one without replacement. Find the probability that none is red.

**Question 2.** **(2)**

a). Differentiate the following function with respect to 'x' using a suitable

substitution: $f(x) = \sin^{-1}\sqrt{\frac{1 - x}{2}}$ , $0 < x < 1$

**OR**

b). Find the interval in which the function $f(x) = 2 x^3 - 9x^2 + 12x + 15$ is decreasing.

**Question 3.** **(2)**

Show that the function $f : R \rightarrow R$ defined by $f(x) = \frac{x}{x^2 + 1}$ is not onto.

**Question 4.** **(2)**

Using L' Hospital's rule evaluate : $\underset{x \rightarrow 0}{Lt} \frac{\cot x}{\cot 2x}$

**Question 5.** **(2)**

a) Evaluate : $\int \sec^4 x \tan x \, dx$

**OR**

b) Evaluate : $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$

**Question 6.** **(2)**

Find the differential equation of the family of curves represented by

$x^2 + y^2 = ax^3$ where 'a' is a parameter.

**Question 7.** **(4)**

Prove that $\cos^{-1}(63/65) + 2\tan^{-1}(1/5) = \sin^{-1}(3/5)$

**Question 8.** **(4)**

If $y = \cos(\sin x)$, show that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$

**Question 9.** **(4)**

a) Solve : $(x - \sin y)\,dy + \tan y \, dx = 0$

**OR**

b) Evaluate : $\int_{0}^{\pi/2} (2 \log \operatorname{Sin} x - \log \operatorname{Sin} 2x)\, dx$

**Question 10.** **(4)**

a) Urn I contains 2 white and 3 black balls, urn II contains 4 white and 1 black ball and urn III has 3 white and 2 black balls. An urn is selected at random and a ball is drawn at random from it. What is the probability that it is a white ball ?

**OR**

b) A bag has 5 white and 3 black balls. Four balls are successively drawn out one after the other (i) without replacement (ii) with replacement .
What will be the probability that they are alternately of different colour in each case (i) and (ii) ?

**Question 11.** **(6)**

a) Solve the following system of equations using matrix method :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

**OR**

b) Solve the matrix equation :

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$$

**Question 12.** **(6)**

a) Evaluate : $\int \frac{\sin^{-1} x}{x^3} \, dx$

**OR**

b) Evaluate : $\int_{\pi/4}^{3\pi/4} \frac{x}{1 + \sin x} \, dx$

**Question 13.** **(6)**

Show that the volume of the greatest cylinder which can be inscribed in a right circular cone of height 'h' and semi - vertical angle $45^0$ is $\frac{4}{27} \pi h^3$

**Question 14.** **(6)**

A class consists of 50 students out of which there are 10 girls. In the class 2 girls and 5 boys are rank holders in an examination. A student is selected at random from the class and is found to be a rank holder. What is the probability that the selected student is a girl ?

## SECTION-B (15 marks)

**Question 15** **[5x1]**

In sub-parts (i) and (ii) choose the correct options and in sub-parts (iii) to (v) answer the questions as instructed.

**i)** If the position vectors of the points P and Q are $2\hat{i}+\hat{k}$ and $-3\hat{i}-4\hat{j}-5\hat{k}$ respectively, then vector $\overrightarrow{QP}$ is

a) $5\hat{i} + 4\hat{j} +4\hat{k}$
b) $5\hat{i} +4\hat{j} +6\hat{k}$
c) $5\hat{i} -4\hat{j} +4\hat{k}$
d) $-\hat{i} -4\hat{j} -4\hat{k}$

**ii)** The x-co-ordinate of a point on the line joining the points A(2,2,1) and B(5,1,-2) is 4. Then the z-co-ordinate of the point is

a) 1
b) -4
c) -1
d) 4/3

**iii**) If the vectors $3\hat{i} - 2\hat{j} + m\hat{k}$ and $-2\hat{i} + \hat{j} + 4\hat{k}$ are perpendicular to each other, then find m.

**iv)** Find the sine of the angle between $\vec{a}$ and $\vec{b}$ if $|\vec{a}|=2$, $|\vec{b}|=5$, and $|\vec{a} \times \vec{b}|=8$.

**v)** Find the intercepts made by the plane $2x - y + 3z = 6$ on the co-ordinate axes.

**Question 16** **[2]**

**a)** If $\vec{p}$ is a unit vector and $(\vec{x} - \vec{p}).(\vec{x} + \vec{p})=8$, then find $|\vec{x}|$.

**OR**

**b)** Find a unit vector perpendicular to both the vectors $2\hat{i} - 3\hat{j} + 6\hat{k}$ and $3\hat{j} - 4\hat{k}$.

**Question 17** **[4]**

**a)** Find the vector equation of the plane passing through the point (2, -1, 5 ) which is perpendicular to the plane $x + 2y - 3z = 7$ and parallel to the line $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$

**OR**

**b)** If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of 'k'.

**Question 18** **[4]**

Draw a rough sketch and find the area of the region bounded by the curve

$x = 4y - y^2$ and the y - axis.

## SECTION-C (15 marks)

**Question 19** **[5x1]**

In sub-parts (i) and (ii) choose the correct options and in sub-parts (iii) to (v) answer the questions as instructed.

**i)** If the cost function of a firm is given by $C(x) = 2x^2 - 4x + 5$, then the marginal cost when $x = 10$ is

a) Rs. 36
b) Rs. 10
c) Rs. 18
d) Rs. 20

**ii)** Given the following data regarding variables $'x'$ and $'y'$ :

Regression co-efficient $b_{xy} = 2/9$, Regression co-efficient $b_{yx} = 2$

Variance of $x = 16$, Correlation co-efficient $\gamma(x, y) = 2/3$

Then the value of $\sigma_y$ (standard deviation of y ) is

a) 4/3
b) 12
c) 48
d) None of the above

**iii)** A firm produces x units of output at a total cost of Rs. $\frac{x^3}{3} - x^2 + 5x + 3$ . Find the average variable cost.

**iv)** The revenue function is given by $R(x) = 1000 + 500x - 5x^2$ . Find the quantity at which marginal revenue is zero.

**v)** The two lines of regression are given as $2x + 3y - 10 = 0$ and $4x + y - 5 = 0$. Find mean of $'x'$ and mean of $'y'$ .

**Question 20** **[2]**

**a)** A television manufacturer finds that the total cost for producing and marketing x television sets is $C(x) = 250x^2 + 3250x + 10000$ . Each product is sold for Rs. 6500 . Determine the profit function and breakeven points.

**OR**

**b)** The average cost function for a commodity is given by $AC = x + 5 + \frac{36}{x}$ in terms of output x. Find the output for which AC increases.

**Question 21** **[4]**

**a)** The two lines of regression are given as $2x + y = 7$ and $x + 3y = 11$ . Identify which equation is 'x' on 'y' and which one is 'y' on 'x'. Also find regression co-efficients $b_{xy}$ , $b_{yx}$ and correlation co-efficient $\gamma(x, y)$.

**OR**

**b)** Given the following data for two variables 'x' and 'y':

| | | | | | |
|---|---|---|---|---|---|
| x | 15 | 12 | 11 | 14 | 13 |
| y | 25 | 28 | 24 | 22 | 30 |

Estimate the value of 'y' given x=10 using the appropriate line of regression.

**Question 22** **[4]**

Two tailors P and Q earn Rs. 150 and Rs.200 per day respectively. P can stitch 6 shirts and 4 trousers per day while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum labour cost? Formulate the above as an L.P.P. and solve it graphically. Indicate the feasible region on the graph.

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